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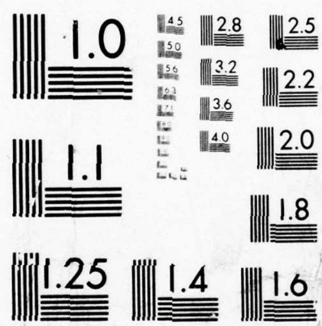

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A MATRIX APPROACH
TO A PROPAGATION CODE

THESIS

GEO/PH/77-1 Peter L. Misuinas
2nd Lt USAF

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A MATRIX APPROACH
TO A PROPAGATION CODE

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by
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December 1977

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Peter L. Misuinas

Contents

<u>List of Symbols</u>

a = beam radius ($1/e^2$ point) in m

A = water vapor pressure in mm Hg

$b(z)$ = aerosol scattering term

B = vertical water vapor coefficient in km^{-1}

c_p = specific heat at constant pressure in $J/g\,^oK$

C_N^2 = refractive index structure constant in $m^{-2/3}$

d = gas mass density in g/m^3

h = beam altitude in m

H = beam altitude in km

I = beam intensity in w/m^2

k = thermal conductivity in $J/m\text{-}sec\,^oK$

L = total length of propagation path in m

n = index of refraction

N = number of propagation steps in beam path

P = beam power in w

q = turbulence parameter in m

r = beam radius in m

s = mean distance from transmitter for each propagation step in m

T = temperature in ^oK

v = wind velocity in m/sec

x = beam radius along X direction in m

z = propagation step distance in m

α = absorption coefficient in km^{-1}

β = thermal blooming term

λ = wavelength in m

ϕ = elevation angle of laser beam in degrees

θ = beam divergence in radians

ω = angular velocity of transmitter

Abstract

A transfer matrix for CO_2 laser beams with an assumed gaussian intensity distribution is developed that includes parameters for absorption, turbulence, and thermal blooming. Calculated parameters are an effective beam radius ($1/e^2$ point) and an on-axis intensity. For moderate power levels, results are consistent with the computer code COMBO. The blooming model predicts results worse than those predicted by COMBO for high power levels.

54

A MATRIX APPROACH TO A PROPAGATION CODE

I. Introduction

The primary concern in nearly all high energy laser applications is the fraction of transmitted power that reaches the receiver. Calculating this fraction for atmospheric transmission is difficult due to the inhomogenity of the atmosphere as a medium, and to the modifications caused by the passage of the laser beam itself. In addition, various transmitter/receiver scenarios, such as ground to air, air to air, stationary to moving, etc., will affect the received power.

Thus there exists a need for a method of easily determining how efficiently a given laser beam will pass through the atmosphere.

Background

There are in existence several computer codes that will calculate how well a laser beam will propagate through the atmosphere. However, nearly all these codes are long and cumbersome, requiring large amounts of computer storage and computer time, or are based on data tables derived from experimental observations, again requiring large storage capability.

Both code types require a computer to operate, are not readily available, and are not easily portable. Also,

tabular codes, while perhaps useful, lack an apparent basis in physics.

Problem

The purpose of this research is to develop a fast, easy to apply method of calculating laser transmission that does not necessarily require a computer. This method, or code, is to have a basis in physics and is designed to determine the on-axis intensity and radius of the laser beam at the receiver when given an initial transmitted power and aperture radius.

Scope and Assumptions

The following code is designed primarily for continuous wave CO_2 (10.6 um) laser beams. Only continuous wave beams are considered so that the beam can be assumed to be in steady state conditions. The areas of the propagation problem included here are absorption, turbulence, and thermal blooming. A linear scattering term is included in the absorption section. The thermal blooming expression has a correction factor for transmitter rotation (beam slewing) and wind.

There are three primary assumptions made. The laser beam is assumed to have a gaussian intensity distribution at all times. The atmosphere is modeled as a series of homogeneous layers, each layer having a thickness small compared with the total path length. Finally, changes in beam direction are assumed to be small so that paraxial approximations and geometrical optics are valid.

Approach

An approach based on ray transfer matrices is used. In geometrical optics a light ray can be traced through a system using a transfer matrix derived from the system characteristics (Ref 4:171-175). Used for years in designing lenses and lens systems, matrix techniques have not been applied previously to laser transmission.

In this paper a separate matrix is developed for each part of the propagation problem. The three individual matrices are then linearly combined into one overall transfer matrix. This final matrix is used to transfer the laser beam through the atmospheric layers having predetermined parameters (absorption coefficients, wind velocities, altitude, width).

There are several advantages to this approach. Matrices are easy to use, especially in repeated applications of the same equations. If one area of the propagation problem is of no interest, the matrix for that area may be deleted or modified without affecting the other matrices. In addition, beam intensity and size can be determined at any point along the propagation path.

To determine if calculations using the matrix code are reasonable, matrix results are compared with results from the computer code COMBO (Ref 6). COMBO appears to be an aggregate code based on derived equations and also tables and curves formed from experimental data and observations.

II. Theory and Development

Matrix Optics

As an example of matrix optics, a ray transfer matrix is formulated for the system shown below.

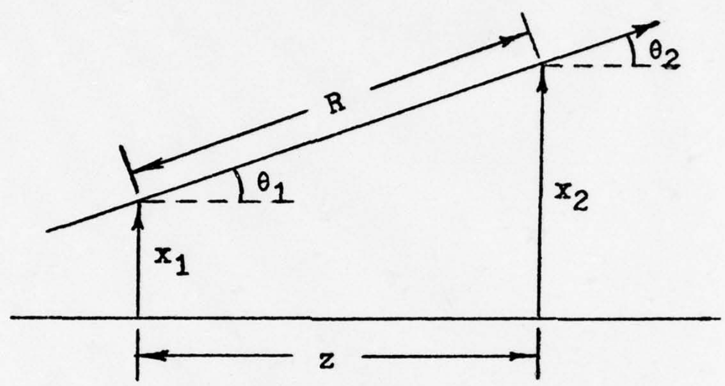

Fig. 1. Ray Passing Through a Medium

For this system, x_1 and x_2 are the initial and final radii, and θ_1 and θ_2 are the initial and final divergence angles, respectively.

The value for x_2 can be calculated from the equation

$$x_2 = x_1 + R(\sin\theta_1) \tag{1}$$

However, if θ is small, the paraxial approximations of $\sin\theta \approx \theta$ and $R \approx z$ may be used, and equations for the system can be written as

$$x_2 = x_1 + z\theta_1 \tag{2}$$

$$\theta_2 = (0)x_1 + \theta_1 \tag{3}$$

since θ does not change for this particular system. If a matrix format is used, the equations can be written as

$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} 1 & z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} \tag{4}$$

or, in a similar way, equations can be derived for any system or medium and represented in the form

$$\begin{bmatrix} x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} C & D \\ E & F \end{bmatrix} \begin{bmatrix} x_1 \\ \theta_1 \end{bmatrix} \tag{5}$$

which is the standard ray transfer matrix for geometrical optics. In this manner the off-axis position (radius) and divergence of a beam can be determined for any system, if θ is small. Since lasers have beam divergences on the order of milliradians, this assumption is valid.

However, the standard matrix contains no provision for determining beam intensity, or equivalently, beam power. To represent power loss, it is necessary to use a matrix of the form

$$\begin{bmatrix} P_2 \\ x_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} P_1 \\ x_1 \\ \theta_1 \end{bmatrix} \tag{6}$$

where the matrix elements A through I must now be determined.

Absorption Matrix

Linear absorption losses for CO_2 laser beams are primarily due to aerosol absorption and scattering, water absorption,

and CO_2 absorption. For a given path length between initial and final positions of (N - 1) and N, respectively, the final beam power is calculated from

$$P_N = P_{N-1}\exp[-z(\alpha_s + \alpha_w + \alpha_c)] \qquad (7)$$

where α_s is the absorption due to aerosols, α_w is the absorbtion due to water vapor, and α_c is the absorption due to atmospheric CO_2. The units of α are usually km^{-1}.

The effects of aerosol scattering and absorption are strongly dependent on local atmospheric conditions. Location, particle size, particle density, and other variables contribute to aerosol loss. See, for example, W. A. Proctor's article, "A Short Descriptive Survey of Atmospheric Aerosols" (Laser Digest, June 1963). A general model is beyond the scope of this paper, but average sea level coefficients of $\alpha_s = 0.015$ km^{-1} and $\alpha_s = 0.073$ km^{-1} for clear (23 km visibility) and hazy (5 km visibility) conditions, respectively, may be used (Ref 2:1486). For high altitude paths or visibility greater than 25 km, $\alpha_s \approx 0$ is a reasonable approximation.

For later calculations, it is convenient to define an aerosol coefficient b(z) such that

$$b(z) = \exp(-\alpha_s z) \qquad (8)$$

where b(z) is unitless.

The water vapor coefficient α_w is also calculated in units of km^{-1}. For horizontal beam paths, α_w as a function of altitude H may be calculated from

TABLE I

VALUES OF A AND B

Humidity (%)	Summer (B = 0.505) (T = 30°C) A (mm Hg)	Winter (B = 0.575) (T = 25°C) A (mm Hg)
10	3.28	2.43
20	6.56	4.85
30	9.85	7.28
40	13.13	9.70
50	16.41	12.12
60	19.69	14.55
70	23.78	16.98
80	26.26	19.40
90	29.54	21.83

$$\alpha_w(H) = 8.34\text{x}10^{-4}(3.94A\exp[-H(B + 0.13)] + A^2\exp[-2BH]) \quad (9)$$

where H is in km (Ref 5:1474-1476). The constants in equation (9) are derived from experimental measurements made by the authors of Ref 5. The value of A is the water vapor pressure at ground level in mm Hg and is a function of temperature and humidity. B is the exponential coefficient of the change in water vapor pressure with altitude and has the units of km^{-1}. B is assumed to be a constant for summer and winter. Some values for A and B are listed in Table I. See Appendix A for further explanation.

For beam paths that change altitude, equation (9) is integrated over the change in altitude to give α_w as a function of final altitude H_2 and initial altitude H_1. The result is

$$\alpha_w(H_2,H_1) = \frac{4.17\text{x}10^{-4}A^2}{B}(\exp[-2BH_1] - \exp[-2BH_2]) +$$
$$\frac{3.28\text{x}10^{-3}A}{B + 0.13}(\exp[-H_1(B + 0.13)] - \exp[-H_2(B + 0.13)]) \quad (10)$$

where $\alpha_w(H_2,H_1)$ still has the units of km^{-1} (Ref 5:1476).

For atmospheric CO_2 absorption, the absorption coefficients determined by Yin and Long are used (Ref 9). Values of α_c as a function of altitude may be found from the graph in Fig. 2. Appendix B contains a list of polynomials used to plot Fig. 2. For beam paths that change altitude, the polynomials listed in Appendix B are integrated over the change in altitude to give α_c as a function of (H_2,H_1). Table II lists some values of $\alpha_c(H_2,H_1)$.

The values of $\alpha_w(H_2,H_1)$ and $\alpha_c(H_2,H_1)$ as previously

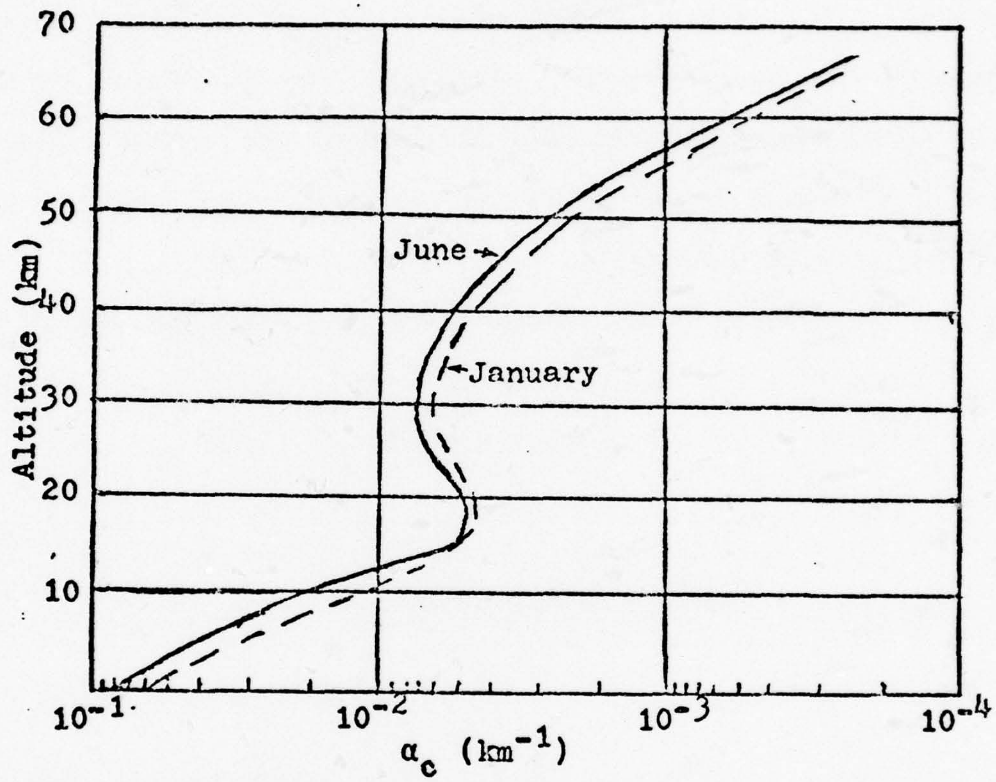

Fig. 2. CO_2 Absorption Coefficient versus Altitude

TABLE II

CO_2 ABSORPTION COEFFICIENTS

H_1 to H_2 (km)	January α_c (km^{-1})	July α_c (km^{-1})
0-1	0.060	0.079
0-2	0.115	0.148
0-3	0.165	0.208
0-4	0.207	0.261
0-5	0.242	0.307
0-6	0.272	0.347
0-7	0.297	0.381
0-8	0.318	0.409
0-9	0.335	0.433
0-10	0.348	0.451
0-12	0.367	0.477
0-20	0.408	0.520

(From Ref 9:1553)

defined are valid only for vertical paths. For slant paths, the equation for the total absorption coefficient α in km^{-1} is

$$\alpha = \csc\phi[\alpha_w(H_2,H_1) + \alpha_c(H_2,H_1)] \qquad (11)$$

where ϕ is the elevation angle with respect to the horizon of the laser beam. See Fig. 3.

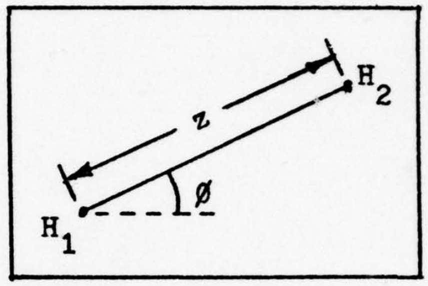

Fig. 3. Beam Traversing a Slant Path

Thus the final expression for absorption, including aerosol scattering, is

$$P_N = b(z)P_{N-1}\exp(-\alpha z) \qquad (12)$$

where z is in km. The transfer matrix for absorption only is

$$\begin{bmatrix} P_N \\ x_N \\ \theta_N \end{bmatrix} = \begin{bmatrix} b(z)\exp(-\alpha z) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{N-1} \\ x_{N-1} \\ \theta_{N-1} \end{bmatrix} \qquad (13)$$

Turbulence

Atmospheric turbulence is modeled as a series of homogeneous bubbles, or turbules, of varying refractive indices. As the beam passes through the turbules, it is refracted, and the net effect is to increase the beam radius.

The increase in radius due to turbulence may be calculated from

$$q = \frac{z\lambda}{p\pi} \qquad (14)$$

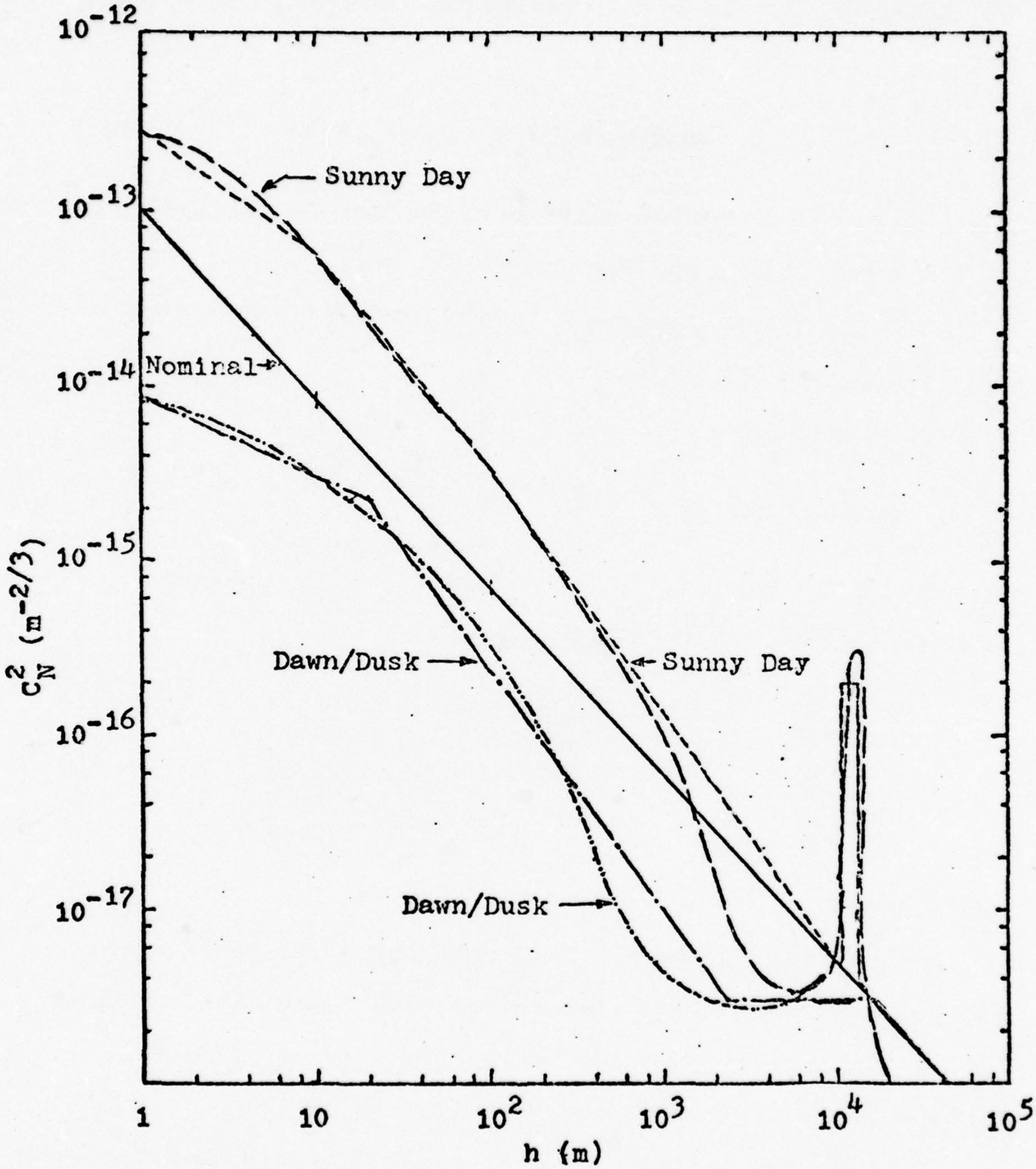

Fig. 4. Refractive Index Structure Constant vs Altitude

where q, z, λ, and p have the units of m. The value for p is calculated from the equation

$$p = [57.24\lambda^{-2}\sec\phi\int_s^t C_N^2 (\frac{t-l}{L})\, dl]^{-3/5} \qquad (15)$$

where C_N^2 is the refractive index structure constant, s and t are the initial and final path points, and $L = t - s$ is the total length of the propagation path in m (Ref 10:2771). The refractive index structure constant C_N^2 is a measure of how the refractive index varies due to turbulence as a function of altitude. Several plots of C_N^2 are shown in Fig. 4.

Since C_N^2 is not a function of path length, equation (15) can be integrated to give

$$p = (21.47\lambda^{-2} L\, C_N^2 \sec\phi)^{-3/5} \qquad (16)$$

An approximate expression for C_N^2 as a function of h is

$$C_N^2(h) = 10^{-13} h^{-1.075} \qquad (17)$$

where h is the mean altitude in m of the beam path (Ref 6: 20). This is equivalent to letting h be represented by

$$h = h_o + \frac{L\sin\phi}{2} \qquad (18)$$

where h_o is the altitude of the transmitter and is set equal to 1 m for ground level. Combining equations (14) through (18) gives the expression

$$q = \frac{2.00\,z}{\lambda^{1/5}} \left[\frac{L \sec\phi \times 10^{-13}}{(h_o + \frac{L \sin\phi}{2})^{1.075}} \right]^{3/5} \tag{19}$$

Equation (19) is independent of the matrix parameters, but a division by the beam radius a allows the transfer matrix for increases in the radius only to be written as

$$\begin{bmatrix} P_N \\ x_N \\ \theta_N \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 + \frac{q}{a} & z \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_{N-1} \\ x_{N-1} \\ \theta_{N-1} \end{bmatrix} \tag{20}$$

Thermal Blooming

Thermal blooming is a non-linear effect associated with high energy laser beams. For CO_2 laser beams it is mainly a result of power absorption by atmospheric CO_2 and water vapor. The absorbed power heats the atmosphere and causes the air density to decrease at the beam center. As a result the atmosphere acts like a negative lens, spreading the beam and reducing on-axis intensity.

The steady-state energy equation describing the power absorption is

$$dc_p v \frac{\partial T}{\partial x} - k\nabla^2 T = \alpha I \tag{21}$$

where d is the gas mass density in g/m^3, c_p is the specific heat at constant pressure in $J/g\,^{o}K$, v is the wind velocity in m/sec in the X direction, k is the thermal conductivity in $J/m\text{-}sec\,^{o}K$, T is the temperature in ^{o}K, and I is the intensity in w/m^2 (Ref 8:601). For no wind (v = 0), the development outlined by Smith (Ref 8) is followed. Letting I =

$I_o \exp(-2r^2/a^2)$ and using the relation

$$\frac{\partial\theta}{\partial z} = \frac{1}{n}\left(\frac{\partial n}{\partial T}\right)\left(\frac{\partial T}{\partial r}\right) \tag{22}$$

the angular divergence due to blooming is

$$\theta_r - \theta_o = -\frac{P(\partial n/\partial T)(1 - \exp[-2r^2/a^2])(1 - \exp[-\alpha z])}{2\pi knr} \tag{23}$$

where θ_r is the beam divergence at radius r, θ_o is the angular divergence without thermal effects, and $(\partial n/\partial T)$ is the variation of the index of refraction with temperature.

Letting the index $n \approx 1.0$ and $(1 - \exp[-\alpha z]) \approx \alpha z$ (valid for $\alpha z \leq 0.1$), the equation becomes

$$\theta_r - \theta_o = -\frac{P\alpha z(\partial n/\partial T)(1 - \exp[-2r^2/a^2])}{2\pi kr} \tag{24}$$

The beam is assumed to retain a gaussian distribution as it propagates. In order to use a ray transfer matrix, the beam is now assumed to be an infinite bundle of rays gaussianly distributed with the radius r. Each ray has a given divergence dependent on r as shown by equation (24), with the beam divergence at $r = 0$ and $r = \infty$ being equal to zero. To avoid the task of tracing each ray, all rays (except at $r = 0$) are now assumed to have some average divergence $\overline{\theta_r - \theta_o}$. Expanding the exponential in equation (24) and assuming beam truncation by the aperture at the exp(-2) point, the average divergence is calculated to be

$$\overline{\theta_r - \theta_o} = -\frac{0.0432P\alpha z(\partial n/\partial T)}{ka} \tag{25}$$

for no wind.

If a wind transverse to the beam path is introduced, the steady-state energy equation is difficuly to solve analytically. In order to avoid numerical computer rechniques, an approximate solution is used.

As the beam is assumed to have a gaussian intensity distribution, it is reasonable to assume the temperature of the beam is also gaussian in nature. If the temperature distribution with x is assumed to be of the form

$$T = T_o \exp(-2x^2/a^2) = \mathrm{EXP} \tag{26}$$

where T_o is the temperature at $x = 0$, then the first and second derivatives are

$$\frac{\partial T}{\partial x} = -\frac{4x}{a^2}\mathrm{EXP} \tag{27}$$

and

$$\frac{\partial^2 T}{\partial x^2} = \left(-\frac{4}{a^2} + \frac{16x^2}{a^4}\right)\mathrm{EXP} \tag{28}$$

The second derivative can now be written in terms of the first derivative as

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial x}\left(\frac{1}{x} - \frac{4x}{a^2}\right) \tag{29}$$

Since the radius of interest is the exp(-2) point, and to simplify matters and produce an equation that can easily be solved, equation (29) is evaluated at the point $x = a$ to give

$$\frac{\partial^2 T}{\partial x^2} = - \frac{3}{a}\left(\frac{\partial T}{\partial x}\right) \tag{30}$$

(Ref 7). Then, letting $(\partial^2 T/\partial y^2) \approx (\partial^2 T/\partial x^2)$, the steady-state energy equation can be solved to give

$$\frac{\partial T}{\partial x} = \frac{\alpha I a}{d c_p v a + 6k} \tag{31}$$

Substituting equation (31) into equation (22), the angular divergence can be solved as before to be

$$\overline{\theta_x - \theta_o} = - \frac{0.1066\, P \alpha z (\partial n/\partial T)}{a(d c_p v a + 6k)} \tag{32}$$

for the angular divergence in the X direction.

Any wind is assumed to reduce blooming only in the wind direction. Neglecting wind steering effects, wind reduces blooming in the X direction and the beam becomes elliptical in shape with an area of $\pi a_x a_y$, where a_x and a_y are the minor and major radii of the ellipse. Total beam power is calculated by integrating over the beam area. To simplify the integration the ellipse is represented by a circle of the same area. The equivalent radius of the circle is

$$a = (a_x a_y)^{1/2} = [(z\overline{\theta_x})(z\overline{\theta_y})]^{1/2} \tag{33}$$

where $\overline{\theta_x}$ and $\overline{\theta_y}$ are the beam divergences in the X direction and the Y direction, respectively, due to thermal effects only. Substituting equations (32) and (25) for $\overline{\theta_x}$ and $\overline{\theta_y}$ gives

$$a = z\bar{\theta} = z\left(\frac{0.068\, P\alpha z(\partial n/\partial T)}{a(kdc_p va + 6k^2)^{1/2}}\right) \tag{34}$$

where the absolute value of $(\partial n/\partial T)$ is used.

Thus the thermal blooming expression is

$$\beta = \frac{0.068\, \alpha z(\partial n/\partial T)}{a(kdc_p va + 6k^2)^{1/2}} \tag{35}$$

and the transfer matrix for calculating beam divergences only is

$$\begin{bmatrix} P_N \\ x_N \\ \theta_N \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \beta & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{N-1} \\ x_{N-1} \\ \theta_{N-1} \end{bmatrix} \tag{36}$$

Complete Transfer Matrix

Adding matrices (13), (20), and (36) gives the overall transfer matrix for absorption, turbulence, and thermal blooming. The final form is

$$\begin{bmatrix} P_N \\ x_N \\ \theta_N \end{bmatrix} = \begin{bmatrix} b(z)\exp(-\alpha z) & 0 & 0 \\ 0 & 1 + \frac{q}{a} & z \\ \beta & 0 & 1 \end{bmatrix} \begin{bmatrix} P_{N-1} \\ x_{N-1} \\ \theta_{N-1} \end{bmatrix} \tag{37}$$

and is the form that should be used for all calculations. If a given area is to be omitted, for example, thermal blooming, setting $\beta = 0$ would give the transfer matrix for absorption and turbulence only.

Given initial values of P_o, x_o, and θ_o, equation (37) is applied N times ($N = L/z$). For reasonable results, N should have a value greater than 10, and is restricted by the

TABLE III

VALUES FOR d AND ($\partial n/\partial T$)

Altitude (km)	$d \times 10^3$ (g/m^3)*	$(\partial n/\partial T) \times 10^{-7}$ ($^{\circ}K^{-1}$)
0.0	1.22	8.84
0.5	1.17	8.52
1.0	1.11	8.21
1.5	1.06	7.90
2.0	1.01	7.61
2.5	0.96	7.32
3.0	0.91	7.04
4.0	0.82	6.50
5.0	0.74	5.99
8.0	0.53	4.63
10.0	0.41	3.85
20.0	0.09	0.85

*d must be changed to units of g/cm^3 when used in equation (40)

earlier assumption that $\alpha z \leq 0.1$. In general, $\alpha z \leq 0.1$ is valid for $z \leq 500$ m.

After applying equation (37) N times, the final beam power P_N, final radius x_N, and final beam divergence θ_N at the receiver are obtained. To get a final on-axis intensity, it is recalled that the total power in a gaussian beam can be calculated by integrating over the volume of the gaussian, or from

$$P_N = \int_0^{2\pi} \int_0^{x_N} I_o \exp(-2r^2/x^2)\, r dr\, d\theta \qquad (38)$$

Where I_o is the on-axis intensity of the beam. Equation (38) can be solved to give

$$I_o = \frac{P_N}{1.36\,(x_N^2)} \qquad (39)$$

which is used to determine the on-axis intensity at the receiver.

Concerning the final transfer matrix, a few comments are necessary. For the constants in the matrix, c_p and k vary little with altitude, and the values $c_p = 1.0$ J/g ^oK and k = 0.024 J/m-sec ^oK may be used at all times. However, the values for d and $(\partial n/\partial T)$ decrease with altitude and should be changed for every kilometer increase in altitude. Table III is a list of some values for d and $(\partial n/\partial T)$. The values for $(\partial n/\partial T)$ are calculated from

$$\frac{\partial n}{\partial T} = \frac{0.21d}{T} \qquad (40)$$

in $^{o}K^{-1}$ (Ref 8:601) where d is in g/cm^3 and T is in ^{o}K (Ref 3:3-70).

Transmitter rotation, if present, is included in the thermal blooming expression by solving for an equivalent wind velocity of

$$v = v_o + \omega s \tag{41}$$

in m/sec where v_o is the transverse wind velocity, ω is the transmitter angular velocity, and s is the mean distance from the transmitter for each propagation step.

TABLE IV

Comparison of Matrix Code and COMBO

Power (kw)	Code	Wind (m/sec) 1.0	10.0	25.0	50.0
5	Matrix	I* = 1.14 x = 0.44	I = 6.07 x = 0.19	I = 12.17 x = 0.13	I = 20.89 x = 0.10
	COMBO	I = 5.44 x = 0.12	I = 14.01 x = 0.12	I = 15.12 x = 0.12	I = 15.28 x = 0.12
10	Matrix	I = 0.88 x = 0.70	I = 4.38 x = 0.32	I = 8.55 x = 0.22	I = 14.35 x = 0.17
	COMBO	I = 6.29 x = 0.26	I = 24.09 x = 0.13	I = 28.73 x = 0.12	I = 30.24 x = 0.12
20	Matrix	I = 0.70 x = 1.12	I = 3.30 x = 0.51	I = 6.26 x = 0.37	I = 10.30 x = 0.29
	COMBO	I = 5.32 x = 0.40	I = 35.88 x = 0.16	I = 51.50 x = 0.13	I = 57.46 x = 0.12
50	Matrix	I = 0.54 x = 2.01	I = 2.38 x = 0.96	I = 4.40 x = 0.70	I = 7.06 x = 0.56
	COMBO	I = 3.03 x = 0.85	I = 54.42 x = 0.20	I = 89.71 x = 0.16	I =120.47 x = 0.13
100	Matrix	I = 0.45 x = 3.12	I = 1.93 x = 1.50	I = 3.50 x = 1.12	I = 5.53 x = 0.89
	COMBO	I = 1.77 x = 1.57	I = 62.91 x = 0.26	I =122.86 x = 0.19	I =179.43 x = 0.16

*I has the units of (10^4) w/m^2
x has the units of m

III. Results and Discussion

Comparison with COMBO

Results calculated using the matrix code were originally intended to be compared with real data collected from controlled experiments. However, such data is nearly non-existent, especially in the open literature. Therefore comparisons were made with the computer code COMBO.

A series of comparisons were made using various initial power levels and wind velocities. The results of the trial cases are listed in Table IV. Except for the cases of high power levels, the results of the two codes are consistent with each other.

Comments

In order to allow a better comparison between the matrix code and COMBO, a single scenario was used for all trial cases.

The problem chosen was the propagation of a laser beam over a path 5 km long at a constant altitude of 61 m. The total absorption coefficient was the same for both codes. Since COMBO does not include scattering, the aerosol scattering coefficient b(z) was set equal to 1.0. In addition, turbulence effects were not included in the trial cases, as the increase in radius due to turbulence is small (0.01 to 2.0 cm), and the two codes predicted nearly identical results

for test cases involving turbulence effects only.

The difference in results is thus due to the thermal blooming models of the two codes. For lower power levels, results predicted by the two codes are comparable. For high power levels, the matrix code results disagree with those of COMBO. The assumption that mass flow in the X direction does not affect blooming in the Y direction is no longer valid.

A more exact solution of the energy state equation would perhaps correct this fault in the matrix model, but the obtainment of that solution would, most likely, increase the complexity of the matrix code to the point where little would be gained over current models in terms of computer time and storage.

In general, the major area of disagreement among all models is how thermal blooming effects are calculated. Certainly the matrix code, if it does not predict actual results, at least calculates the worst case.

The absorption models are for 30° N. latitude. To adapt the water model to other locations requires adjusting the constants A and B in equations (9) and (10). See Appendix A for details. If absorption coefficients are already available, the models may be dropped entirely.

The turbulence model predicted the same results as that of COMBO. Two cases, one at 61 m and one at 10,000 m, were run for turbulence only. Both the matrix code and COMBO calculated a radius increase of 1.8 cm for the first case, and an increase of 0.09 cm for the second case.

Although all the examples presented assumed a CO_2 laser beam, the matrix code need not be restricted to CO_2 lasers. By properly adjusting (or substituting) the absorption coefficients, lasers at any given wavelength may be traced using the matrix formulation developed in this paper.

Suggestions and Recommendations

The CO_2 model is too general for low altitudes. A more complete model based on local conditions is necessary for best results.

The thermal blooming model appears to break down at high power levels. A more exact solution to the energy state equation can be investigated to reduce this problem. Another possibility, one less likely to complicate the model, is the development of a correction term based on mass flow in the X direction that reduces blooming in the Y direction.

The model is presently restricted to a gaussian beam. The scope of the model would be greatly increased if a means of incorporating other beam distributions was developed, possibly by representing other distributions by a sum of gaussian beams.

There is interest in the inverse problem. Given a required intensity at the receiver, it is desirable to know the necessary beam power at the transmitter. If the matrix presented here can be solved for an inverse, the power needed for a specified amount of energy at the receiver could be calculated.

Bibliography

1. Brunt, D. Physical and Dynamical Meteorology (Second Edition). London: Cambridge University Press, 1944.

2. Gebhardt, F. G. "High power laser propagation." Applied Optics, 15:1479-1492 (June 1976).

3. Gray, D. E., editor. American Institute of Physics Handbook (Third Edition). New York: The Colonial Press Inc., 1972.

4. Hecht, E. and A. Zajac. Optics. Reading: Addison-Wesley Publishing Company, 1975.

5. McCoy, J. H., et al. "Water Vapor Continuum Absorption of Carbon Dioxide Laser Radiation near 10 u." Applied Optics, 8:1471-1478 (July 1969).

6. Peckham, L. N. and R. W. Davis. A Simplified Propagation Model for Laser System Studies. AFWL-TR-72-95 (Rev). Kirtland Air Force Base, New Mexico: Air Force Weapons Lab, 1960. AD909426L.

7. Personal communication with Captain Phil Nielsen, instructor at the Air Force Institute of Technology.

8. Smith, D. C. "Thermal Defocusing of CO_2 Laser Radiation in Gases." IEEE Journal of Quantum Electronics, QE-5: 600-607 (December 1971).

9. Yin, P. K. L. and R. K. Long. "Atmospheric Absorption at the Line Center of P(20) CO_2 Laser Radiation." Applied Optics, 7:1551-1553 (August 1968).

10. Yura, H. T. "Atmospheric Turbulence Induced Laser Beam Spread." Applied Optics, 10:2771-2773 (December 1971).

Appendix A

Water Vapor Model

The water vapor model is derived from Ref 5. The model as presented is based on averages and is good for general results, but for more exact results, the model can easily be modified to reflect local conditions.

The authors of Ref 5 use water vapor distributions of

$$WVP = 11.0\exp(-0.575H) \tag{42}$$

for January and

$$WVP = 22.8\exp(-0.505H) \tag{43}$$

for July where WVP is the water vapor pressure in Torr. The vertical water vapor coefficients $B = 0.575\ km^{-1}$ and $B = 0.505\ km^{-1}$ are from plots of ln(WVP) versus altitude and are the slopes of straight lines fitted to data (Ref 5:1475). See Fig. 5. If local data is available, a similar plot can be made and new values of B calculated. If not, the values of B given earlier may be

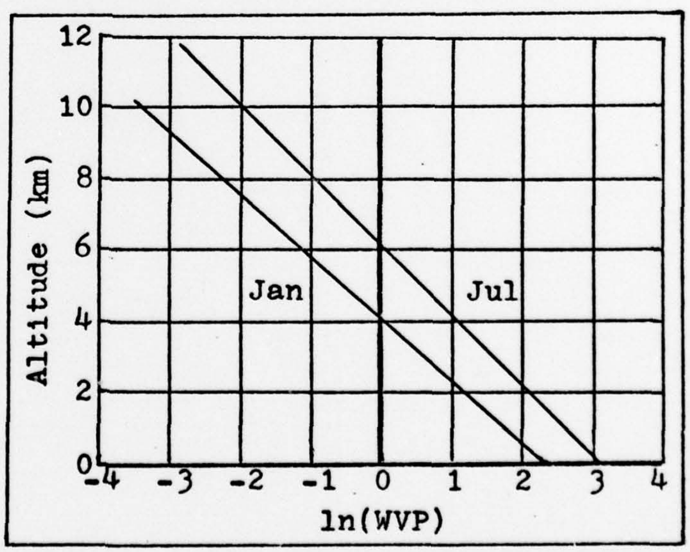

Fig. 5. Water Vapor Distribution Plot (From Ref 5:1475)

used.

The constants A = 11.0 and A = 22.8 Torr are based on the average humidities for January and July. However, for any given temperature and relative humidity, A may be calculated directly from

$$A = 4.58(RH)\exp(5423[\frac{1}{273} - \frac{1}{T}]) \qquad (44)$$

where RH is the relative humidity (RH $\leq$ 1.0) and T is the temperature in ^{o}K (Ref 1:104).

Appendix B

Polynomial Curve Fits For Fig. 2

The following tables contain polynomials that are the result of a curve fitting program for the curves in Fig. 2. The polynomials are used to calculate the CO_2 absorption coefficient α_c for any vertical path beginning at altitude h_1 and ending at altitude h_2, where h_1 and h_2 are in m, by integrating the polynomial (or polynomials) listed for the two altitudes.

The polynomials may also be used to calculate α_c for any given altitude by substituting the altitude h in m directly into the correct polynomial.

TABLE VI

July Polynomial Curve Fit For Fig. 2

Altitude (km)	Polynomial
0-15	$8.585241 \times 10^{-2} - 1.439371 \times 10^{-5}h + 2.255601 \times 10^{-9}h^2 - 3.082077 \times 10^{-13}h^3 + 2.079833 \times 10^{-17}h^4 - 5.087808 \times 10^{-22}h^5$
15-47	$4.409444 \times 10^{-2} - 8.501794 \times 10^{-6}h + 6.408721 \times 10^{-10}h^2 - 2.137128 \times 10^{-14}h^3 + 3.230708 \times 10^{-19}h^4 - 1.912325 \times 10^{-24}h^5$
47-51	$-5.381483 \times 10^{-1} + 3.443774 \times 10^{-5}h - 7.221806 \times 10^{-10}h^2 + 4.994872 \times 10^{-15}h^3$
51-65	$8.595289 \times 10^{-1} - 5.584211 \times 10^{-5}h + 1.371591 \times 10^{-9}h^2 - 1.5063 \times 10^{-14}h^3 + 6.229936 \times 10^{-20}h^4$

h is the altitude in meters (From Ref 9:1552)

TABLE V

January Polynomial Curve Fit For Fig. 2

Altitude (km)	Polynomial
0--2	$6.177301 \times 10^{-2} - 4.243258 \times 10^{-6}h + 3.331791 \times 10^{-11}h^2$
2--12	$7.1772259 \times 10^{-2} - 1.018966 \times 10^{-5}h + 5.438857 \times 10^{-10}h^2 - 1.440797 \times 10^{-14}h^3 + 2.111184 \times 10^{-19}h^4$
12--17	$2.168201 \times 10^{-2} - 1.708914 \times 10^{-6}h + 4.794412 \times 10^{-11}h^2 - 4.821832 \times 10^{-16}h^3$
17--47	$2.749415 \times 10^{-2} - 6.043183 \times 10^{-6}h + 4.952857 \times 10^{-10}h^2 - 1.73234 \times 10^{-14}h^3 + 2.748747 \times 10^{-19}h^4 - 1.644807 \times 10^{-24}h^5$
47--51	$-5.0246274 \times 10^{-1} + 3.209868 \times 10^{-5}h - 6.729368 \times 10^{-10}h^2 + 4.656468 \times 10^{-15}h^3$
51--65	$1.2415226 \times 10^{-1} - 5.636995 \times 10^{-6}h + 8.609204 \times 10^{-11}h^2 - 4.4187473 \times 10^{-16}h^3$

h is the altitude in meters (From Ref 9:1552)

Appendix C

Sample Calculator Programs

The following pages contain two sample calculator programs for a single propagation scenario as an example of how the matrix formulation may be used on a hand calculator. The first program is designed for a Texas Instruments SR 56.. The second program is designed for a Hewlett Packard HP 25.

The propagation scenario that the programs solve is the coaltitude transmission of a laser beam from a transmitter to a receiver over a distance L in propagation steps of distance z. The beam has some initial power P and radius x and is assumed to be focused at a point distance L away.

Because the effect of turbulence is small compared to thermal blooming, it is not included in these programs. If desired, however, it may be added.

SR 56 Program

The memory locations have the following values:

(0) N -- number of propagation steps

(1) P_o-- (0.865)(initial beam power) due to aperture truncation at the exp(-2) point, in watts

(2) x_o-- initial beam radius in meters

(3) θ_o-- initial beam divergence in radians

(4) $-\alpha$-- negative value of total absorption coefficient, km^{-1}

(5) x_o-- initial beam radius in meters

(6) C_1-- constant $C_1 = 0.00068\, z(\partial n/\partial T)$

(7) C_2-- constant $C_2 = 0.024\, vd$

(8) C_3-- constant $C_3 = 6k^2 = 0.003456$

(9) z -- propagation step distance in meters

The program keystrokes are:

LRN	
RCL 3 x RCL 9 + RCL 2 = STO 2	Calculates new radius
RCL 3 - RCL 1 x RCL 4 x RCL 6 ÷((RCL 7 x RCL 5 + RCL 8)$\sqrt{x}$ x RCL 5) = STO 3	Calculates new beam divergence
RCL 2 STO 5	Replaces old value of radius with new
RCL 1 x RCL 4 e^x = STO 1	Calculates new power
2nd dsz 00	Do loop instruction
RCL 1 ÷ (1.36 x RCL 2 x^2) =	Calculates final on-axis intensity
R/S	
RST	
LRN	

Upon pressing the R/S key the program will run to completion, displaying the on-axis intensity at the receiver.

The final beam power is stored in memory location (1) and the final beam radius is stored in memory location (2).

Example: For $N = 50$, $L = 5000$ m, $z = 100$ m, $P = 5000$ w, $h = 61$ m, $x_o = 0.2$ m, $\alpha = 0.07562\ \mathrm{km}^{-1}$, $(\partial n/\partial T) = 8.84 \times 10^{-7}$, $v = 1.0$ m/sec, $d = 1.22 \times 10^{3}\ \mathrm{g/m}^3$, the final values are $P_N = 2960$ w, $x_N = 0.47$ m, and $I = 11400\ \mathrm{w/m}^2$.

The initial values are $P_o = 4325$ w, $x_o = 0.2$ m, and $\theta_o = -4.0 \times 10^{-5}$ radians.

HP 25 Program

The memory locations have the following values:

(0) αz-- total absorption coefficient in km^{-1} x z in km

(1) P_o-- (0.865)(initial beam power) due to aperture truncation at the exp(-2) point

(2) x_o-- initial beam radius in meters

(3) θ_o-- initial beam divergence in radians

(4) 0 -- initial value for do loop

(5) C_1-- constant $C_1 = 0.00068\, z(\partial n/\partial T)$

(6) C_2-- constant $C_2 = 0.024\, vd$

(7) C_3-- constant $C_3 = 6k^2 = 0.003456$

The program keystrokes are:

RCL 1 RCL 0 x RCL 5 x RCL 6 RCL 2 x RCL 7 + $\sqrt{\ }$ RCL 2 x ÷ RCL 3 + $x \rightleftarrows y$	Calculates new divergence angle
RCL 3 STO 3	Stores new divergence angle
CLR X 1 0 0 x RCL 2 + STO 2	Calculates new radius; in place of 100, enter step distance z

RCL 0 CHS e^X RCL 1 x STO 1	Calculates new power
1 STO + 4 RCL 4 5 0 X Y GO TO 01	Do loop instructions; in place of 50, enter the number of propagation steps
GO TO 00	Ends program

Upon pressing the R/S key the program will run to completion, storing the final power in memory (1) and the final radius in memory (2). To calculate the final on-axis intensity, the keystrokes are

RCL 1
RCL 2
x^2
÷
1.36
÷

The display will show the final intensity.

Example: See example for SR 56.

VITA

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A transfer matrix for CO_2 laser beams with an assumed gaussian intensity distribution is developed that includes parameters for absorption, turbulence, and thermal blooming. Calculated parameters are an effective beam radius ($1/e^2$ point) and an on-axis intensity. For moderate power levels, results are consistent with the computer code COMBO. The blooming model predicts results worse than those predicted by COMBO for high power levels.

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